Since $\sinh x = -i\sin ix$, and $\cosh x = \cos ix$, we get, by substituting ix for x in the continued fractions for $\sin x$ and $\cos x$,

$$\sinh x = \cfrac{x}{1 - \cfrac{x^3}{2.3+x^2 - \cfrac{2.3x^2}{4.5+x^2 - \cfrac{4.5x^2}{6.7+x^2 - \text{ etc.},}}}}$$

$$\cosh x = \cfrac{1}{1 - \cfrac{1}{2+x^2 - \cfrac{2x^2}{3.4+x^2 - \cfrac{3.4x^2}{5.6+x^2 - \text{ etc.}}}}}$$

GEOMETRY.

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289 (Incorrectly numbered 288). Proposed by C. N. SCHMALL, College of the City of New York.

From a point P on a given circle to draw two chords such that, (α) chord PA : chord $PB = m : n$ (a given ratio), and, (β) arc PA : arc $PB = 1 : 3$.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let O be the center of the circle, radius r. Also let angle $POA = 2\theta$, angle $POB = 6\theta$. Then $PA = 2r\sin\theta$, $PB = 2r\sin3\theta$.

$$\frac{PA}{PB} = \frac{\sin\theta}{\sin3\theta} = \frac{1}{3-4\sin^2\theta} = \frac{m}{n}. \quad \therefore \sin\theta = \tfrac{1}{2}\sqrt{\frac{3m-n}{m}},\ \sin3\theta = \frac{n}{2m}\sqrt{\frac{3m-n}{m}}.$$

$$\therefore \text{Make angle } POA = 2\sin^{-1}\tfrac{1}{2}\sqrt{\frac{3m-n}{m}};\ \text{angle } POB = 2\sin^{-1}\frac{n}{2m}\sqrt{\frac{3m-n}{m}}.$$

Then chord PA : chord $PB = m : n$; arc PA : arc $PB = 1 : 3$.

290 (Incorrectly numbered 289). Proposed by J. J. QUINN, Ph. D., Scottdale, Pa.

(a) Suppose a circle described around the origin. Then at the end of a uniformly revolving radius r, a line equal to the diameter is pivoted. Find the equation of the locus of its extremity, if for every unit of angle its projection on the X axis is a constant linear unit, being the same part of the diameter as the angle is of π radians.

(b) Show how it can be applied to the trisection or multisection of an angle.

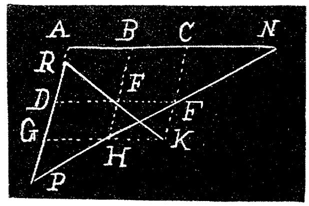
No solution has been received.

292 (Incorrectly numbered 290). Proposed by DR. L. E. DICKSON, The University of Chicago, Chicago, Ill.

Given nine points lying by threes in three columns and in three rows, draw through them, by continuous motion, a broken line composed of only four straight segments, and passing but once through each of the nine points. [A current puzzle.]

Solution by G. I. HOPKINS, A. M., Professor of Mathematics and Astronomy, Manchester (N. H.) High School, and MISS IDA M. SCHOTTENFELS, A. M., New York, N. Y.

Let A, B, C, D, E, F, G, H, and K be the points. It is evident that A, C, K, and G will be the vertices of a parallelogram. Let BH be a median of this parallelogram, and E any point in the median except the center. Then the broken line $ANPRK$ will fulfill the conditions of the problem. This course fails if the middle row and middle column bisect each other. If the row DEF is not parallel to GK, then three lines, or a broken line of three segments will fulfill the conditions of the problem.



G. I. HOPKINS.

If BH and DF are not medians, take the course KER, RP, PN, NB. If BH and DF are medians, take the course KEA, AN, NP, PD; or KEA, AP, PN, NB.

IDA M. SCHOTTENFELS.

Also solved by the Proposer.

296 (Incorrectly numbered 294). Proposed by JOHN JAMES QUINN, Ph. D., Scottdale, Pa.

a) Suppose an indefinite line be pivoted at the end of a revolving radius whose center is the origin; and the initial position of the radius is coincident with the X-axis and the pivoted line perpendicular to it. As the radius revolves through equal amounts of arc the line moves to the right over corresponding equal intercepts on the X-axis. What is the equation of the locus of a point on the line whose distance from the end of the radius is equal to a diameter?

b) Show how the locus can be applied to the multisection of an angle.

c) Suppose the diameter be laid off in both directions.

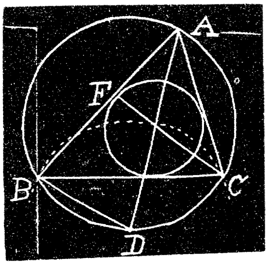
No solution of this problem has been received.

297 (Incorrectly numbered 295). Proposed by S. F. NORRIS, Professor of Mathematics, Baltimore City College, Md.

One side and the opposite angle of a triangle are fixed. Find the locus of the center of the inscribed circle. Solve by methods of analytic geometry.

I. Solution by C. N. SCHMALL, A. B., 89 Columbia Street, New York City.

This problem can be solved more easily and more neatly by Euclidean Geometry. Thus, referring to figure, we have,



$$\angle DCO=\angle DCB+\angle BCO=\angle DCB+\tfrac{1}{2}\angle BCA,$$
$$\angle DOC=\angle AOF=\angle OAC+\angle OCA;$$

but $\angle OAC=\angle DCB$ (since arc $DC=DB$), and $\angle OCA=\frac{1}{2}\angle BCA$. Hence $\angle DCO=\angle DOC$.

Therefore $DC=DO$.

Hence, keeping BC constant and vertex A always on the arc BAC (making opposite angle constant) the locus of center O of the inscribed circle is a circle whose center is D and radius DC.